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The Bakerian Lecture, 1993: Mechanism of Supernovae

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The Bakerian Lecture, 1993
Mechanism of supernovae

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Supernovae of type II happen at the end of the evolution of massive stars, 10 times the mass of the Sun, $M_{\odot}$, or more. To begin with, the central core, of mass about $1.5\,M_{\odot}$, collapses; the large gravitational energy remains for a while in the core. It is then released in the form of neutrinos. A small fraction, 1 or 2%, of the neutrino energy is absorbed in the mantle of the star, i.e. the region 100 or 500 km from the centre; this drives the shock. It is essential that vigorous convection occurs in the shocked material. With reasonable assumptions, one can estimate the energy in the shock to be of the order 10^{51} erg, in agreement with observation. The argument is based on observation and analytical calculations, with a minimum of help from elaborate computations.

1. Importance of supernovae

Supernovae are very spectacular phenomena. In a year, they emit about 10^{51} erg of energy, about the same as the Sun will emit in its lifetime of 10^{10} years. Supernovae of type II occur at the end of the life of massive stars, heavier than about 10 times the mass of the Sun.

More important still, supernovae eject the major part of their mass into space. This contains all the elements which the big progenitor star has manufactured during its lifetime of millions of years, particularly elements heavier than helium, like carbon, oxygen, silicon and iron. The Big Bang only produced hydrogen and helium. Thus supernovae are responsible for nearly all the matter we see around us, including ourselves.

Supernovae have been recorded by the Chinese since antiquity. A specially bright one occurred in 1006, others in 185 and 1054, the latter being the origin of the Crab Nebula. In the West, Tycho Brahe observed one in 1572, Kepler in 1604. In 1987, a supernova was observed in the Large Magellanic Cloud (1987A), close to our own galaxy. About 20 supernovae are observed every year in more distant galaxies.

In massive stars, many nuclear reactions occur in succession, H is converted into He, He into C and O, C into Ne and Mg, O into Si and Si into Fe. The most evolved (and heaviest) atoms are near the centre. Fe is the most tightly bound nucleus and can therefore not generate further nuclear energy. The core of the star no longer has an energy source, its pressure comes only from degenerate electrons (Fermi energy). Once the Fe core exceeds the Chandrasekhar mass of about $1.45\,M_{\odot}$ ($M_{\odot}$ = mass of the Sun), the core will collapse and the supernovae event begins.

The gravitational energy released by the collapse is used to eject the outer parts of the star. This lecture is concerned with the mechanism leading from collapse to ejection.

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The gravitational energy released is about 3×10^{53} erg. The kinetic energy of the ejecta is of order 10^{51}. The energy emitted as light is a few times 10^{49} erg. Most of the gravitational energy is released as neutrinos, a few of which have been observed in the case of 1987A.

2. Sequence of events

The sequence of events is as follows.

1. *Collapse of the core*. The material moves with a velocity which is a substantial fraction, $\alpha = 0.5$–0.7, of the free-fall velocity,

$$v = \alpha(2GM_r/r)^{\frac{1}{2}},$$

where M_r is the mass enclosed by the sphere of radius r. For the most interesting material, M_r is near the mass of the core,

$$M_r \approx 1.5\, M_\odot = 3 \times 10^{33}\ \text{g}, \quad GM_r = 2 \times 10^{26}\ \text{cgs units}.$$

2. *Rebound*. Material outside a 'homologous core' of 0.5–$0.7\, M_\odot$ rebounds. It moves outward with velocities of 10^9 cm s^{-1} and more. A shock is created where the rebounding material meets the infalling matter. Behind (inside) the shock, the temperature is high (more than 1 MeV $\approx 10^{10}$ K). Because of the high temperature, *Fe* is dissociated into nucleons (neutrons and protons), a process which consumes a lot of energy (about 9 MeV per nucleon). This loss of energy usually stops the 'prompt shock' at a distance $r = 200$–500 km from the centre. The material inside the shock comes (more or less) to rest. But material still falls in from large distances, therefore an 'accretion shock' persists, more or less stationary in space.

3. *Neutrino emission*. The infalling material accretes to the core, which may be called a proto-neutron star. As the accreting material hits the dense material of the core, it is stopped, its kinetic energy is converted into internal energy, resulting in a high temperature. This leads to emission of neutrinos. A 'neutrino sphere' is formed, analogous to the photosphere of stars. By definition, the line from the neutrino sphere to the surface of the star is one neutrino mean free path. The neutrinos escaping may then be considered as black-body radiation from the neutrino sphere. Combining the neutrino luminosity with the definition of the neutrino sphere, one finds that the black body temperature is 4–5 MeV, and the material density about 2×10^{11} g cm^{-3}.

Some neutrinos also come from the very interior of the star, but they are emitted somewhat later (about 10 s).

4. *Shock revival*. As suggested by James Wilson in 1982, the region of the star between about 100 and 500 km absorbs neutrinos from the core although only about 1% of them. This revives the shock and supplies its kinetic energy.

5. *Convection*. Neutrino absorption establishes a negative entropy gradient in the mantle of the star. This causes convection which helps the energy production.

Convection occurs simultaneously with accretion, the latter being necessary to provide prompt neutrinos. Figure 1 shows a schematic of the flow of matter.

3. Temperature distribution after collapse

It is possible to obtain an idea of the temperature distribution outside the neutrino sphere. At the important times, after the start of the prompt shock, the infalling material at $r < 1000$ km (and outside the neutrino sphere) has initially been at large

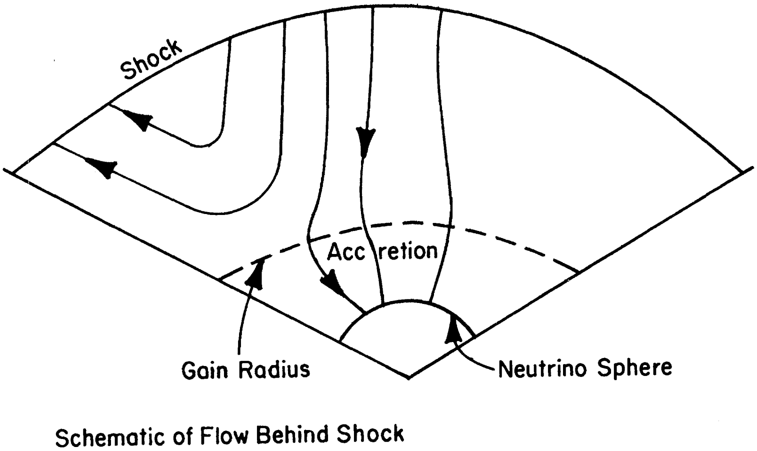

Figure 1. Schematic of the flow in a sector of the supernova. Material comes in through the shock wave. Some of it flows down and accretes to the proto-neutron star. Other parts are heated sufficiently to turn around and flow out again. The dotted line is the gain radius, outside of which the energy gain from neutrino absorption is greater than the loss due to electron capture.

distances, $r > 5000$ km. There the gravitational potential is small, and the internal (including thermal) energy is likewise. The total energy of a mass element is nearly conserved as it falls to the centre, meaning that the total work done by and on the element is small. This is confirmed by numerical computation.

As the material falls in, the absolute value of its gravitational potential,

$$-V_g = GM/r, \tag{1}$$

increases. Gravitational energy is converted into internal and kinetic energy. These two parts are roughly equal outside the shock, but behind (inside) the (accretion) shock, the velocity and hence the kinetic energy is much diminished, hence nearly all the energy is internal. High internal energy and temperature T leads into dissociation of the nuclei originally present, which are likely to be ^{16}O. With increasing T, they first dissociate into ^4He, then into nucleons.

We are especially interested in free nucleons, because only in these can either electrons or neutrinos be captured easily. Equilibrium between nucleons and α-particles, according to the Saha equation and at the expected density (about 10^8 g cm^{-3}) occurs at $kT \approx 1$ MeV. The required energy per nucleon is then:

(*a*) to split ^{16}O into H + n 8.0 MeV
(*b*) to give the nucleons a temperature of 1 MeV 1.5 MeV
(*c*) to give the electron–positron gas the same temperature about 1.1 MeV
giving a total of 10.6 MeV.

This dissociation of oxygen will therefore require the gravitational potential (1) to be

$$-V = 10.6 \text{ MeV/nucleon} = 10.1 \times 10^{18} \text{ erg g}^{-1}. \tag{2}$$

Setting the mass inside the sphere r equal to $1.65\,M_\odot$, the result of computations by Wilson & Mayle (1989), we find that (2) is fulfilled at

$$r = R_\alpha = 207 \text{ km}. \tag{2a}$$

Half the α-particles are dissociated into nucleons at

$$R_{\frac{1}{2}} = 300 \text{ km}. \tag{2b}$$

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In the nucleon region, we can estimate the temperature distribution once we make an additional assumption. This is possible once convection has set in (stage 5 above). Convection tends to make the entropy uniform in space. Assuming this and hydrostatic equilibrium, i.e.

$$-\frac{1}{\rho}\frac{\partial P}{\partial r}=\frac{GM_r}{r^2}, \tag{3}$$

we can calculate the temperature distribution in the nucleon region.

In the nucleon and in the α-particle region, electrons and positrons form a degenerate relativistic gas, with the electrons being slightly more numerous. The difference between their densities is equal to the density of protons. For this case, Bludman & Van Riper (1978) have given formulae which give energy, pressure and entropy of the electron–positron gas in terms of two quantities, the temperature and the electron chemical potential μ_e: the theory is good for temperatures $T > 500$ keV. Surprisingly, the theory also gives the density of nucleons

$$\rho = 0.72\times 10^8 T^3 \eta Y_{\mathrm{P}}^{-1} \tag{4}$$

where T is the temperature in MeV, Y_{P} is the fraction of nucleons (bound or free) that are protons which is about 0.48, and $\eta=\mu_e/T$. From the computations of Wilson & Mayle, I deduce $\eta = 1.05$ so that

$$\rho = 1.57\times 10^8 T^3. \tag{5}$$

This makes it possible to calculate the total pressure, no matter how much of it is due to nucleons and how much to electrons, positrons and electromagnetic radiation. This can then be used in the hydrostatic equation (3) which can be integrated to give

$$T = 2.89/r_7 - 0.39. \tag{6}$$

Here T is the temperature in MeV and r_7, the radius in units of 100 km. The remarkable simplicity of this formula is due to the fact that the ratio of pressure to density is proportional to T, no matter whether the pressure is due to nuclei or light particles. The first term of (6) comes from the integration of (3), the second term is obtained by comparing (6) with the temperature in the mixed region of α-particles and nucleons.

Equation (6) holds only in a very restricted region, namely where the material is mainly nucleons, and where at the same time the entropy is constant, due to convection. But this is just the region where most of the useful neutrino absorption takes place.

4. Neutrino luminosity

We need to know the neutrino luminosity. This has been computed by James Wilson & Ronald Mayle of the Livermore National Laboratory. But it has also been observed (for 1987A) at two laboratories, at Kamiokande in Japan (KII) and at IMB in Ohio. They observed, respectively, 12 and 8 neutrinos; more precisely, positrons produced in the detector by antineutrinos. The detectors consisted of ordinary water and phototubes to observe the Cherenkov light produced by the positrons. KII had about 2000 t of water, IMB 5000, but KII could detect positrons down to about 7 MeV, IMB only down to 20 MeV.

The combined result is shown in the histogram of figure 2 which gives the cumulative number of counts as a function of time. The histogram appears to show

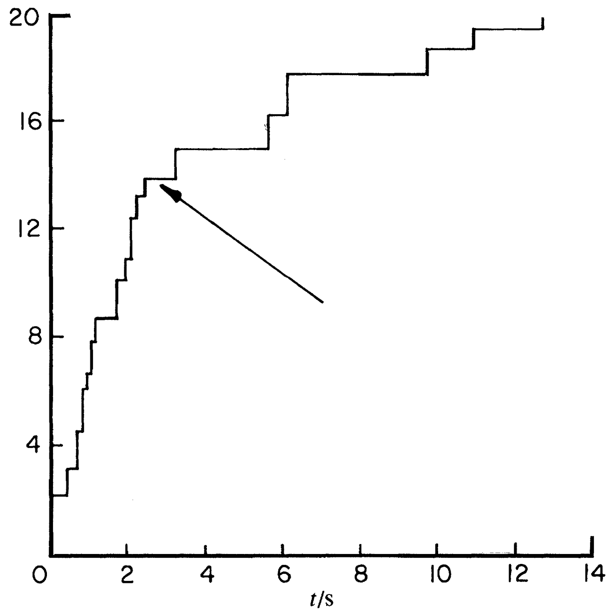

Figure 2. Histogram of the neutrinos from supernova 1987A detected at Kamiokande II and IMB. The arrow shows a point (at about 2.5 s) where the neutrino flux apparently decreases strongly.

a break at 2 or 2.5 s. I interpret this as showing that in the first 2 s there is accretion, leading to very high neutrino luminosity, in the following 10 s we see the neutrinos which have diffused out from the centre of the core.

We use only the KII observations because IMB had too high an energy threshold. Correcting for detector efficiency, KII would have observed 11 anti-neutrinos in the first 2 s if it had had 100% efficiency. Using the effective volume of the counter, and the distance of the Large Magellanic Cloud (50 kilo-parsecs), I find for the neutrino luminosity of 1987A (in $\nu_e + \bar{\nu}_e$)

$$L = 3 \times 10^{52}\ \text{erg s}^{-1}. \tag{7}$$

Electron neutrinos are absorbed by matter with an absorption coefficient,

$$A = 2.7 \times 10^{-20}\, \epsilon^2\ \text{cm}^2\ \text{g}^{-1}, \tag{8}$$

where ϵ is the neutrino energy in MeV. Here it has been taken into account that neutrinos ν_e can only be absorbed by neutrons, anti-neutrinos $\bar{\nu}_e$ only by protons, and it has been assumed that there are equal numbers of each type of nucleons. The average of ϵ^2 can be measured in the Kamiokande experiment, or computed as by Wilson & Mayle (1989). Experiment and calculation agree in giving

$$\langle \epsilon^2 \rangle = 250\ \text{MeV}^2, \tag{8a}$$

so

$$A = 7 \times 10^{-18}\ \text{cm}^2\ \text{g}^{-1}. \tag{8b}$$

5. The gain radius

Matter gains energy by absorbing neutrinos, but it loses energy by capturing electrons or positrons because such capture leads to emission of neutrinos which generally escape from the star. The energy loss by electron capture is

$$\Lambda = 2.0 \times 10^{18} T^6 \Phi(\eta) \quad \text{erg g}^{-1}\ \text{s}^{-1}, \tag{9}$$

where $\Phi(\eta)$ depends slightly on material density, and is about 1.5 in our case. The energy gain by neutrino absorption is

$$G = LA/4\pi r^2 \quad \mathrm{erg\ g^{-1}\ s^{-1}}. \tag{10}$$

The factor T^6 in (9) decreases much faster with increasing r than the gain (10). There will therefore be a radius R_g such that $G > \Lambda$ for $r > R_g$. We call R_g the gain radius, it is defined by $G = \Lambda$. Outside the gain radius, we have a net energy gain. This gain is largest at some point R_m close to R_g; the entropy will have a maximum at R_m. Therefore, we have a negative entropy gradient,

$$\mathrm{d}S/\mathrm{d}r < 0 \quad \text{for} \quad r > R_m.$$

Such a gradient leads to convection. The tendency to convection is further enhanced by the fact that a He region lies above the nucleon region: the nuclear part of the entropy, at given ρ and T, is about the same per particle no matter what the mass of the particle. A He nucleus has about the same entropy as a nucleon, hence the entropy per unit mass decreases strongly in going from nucleons to He.

Convection transports energy, hence all the net energy deposited beyond the entropy maximum will be distributed over the entire region between R_m and the shock. The entropy tends to be equalized over this region. Essentially all the net energy deposited beyond the gain radius becomes energy of the shock.

To calculate the gain radius, we equate $G = \Lambda$. This yields

$$(T^3 R_7)_g = 4.2\sqrt{L_{52}}, \tag{11}$$

where L_{52} is the neutrino luminosity in units of 10^{52} erg s^{-1}, and R_{7g} is the gain radius in units of 10^7 cm. Inserting the observed luminosity and the relation (6) between temperature and radius, we find

$$T_g = 1.73, \quad R_g = 136\ \mathrm{km}. \tag{12}$$

It is important that the temperature T_g is much greater than 1, because at $T = 1$ about half the nucleons combine into He. But He absorbs neutrinos only weakly because to do so, the neutrino would need to have very high energy (greater than 25 MeV). For the same reason, He captures thermal electrons hardly at all.

Neutrino absorption and electron capture inside the gain radius are irrelevant. As long as accretion persists, the energy gained or lost is swept down into the neutron star. When there is no accretion, after a short time, energy gain and loss must balance in each matter element, so (11) holds also for $r < R_g$ and determines $T(r)$.

Therefore processes which happen only very close to the neutrino sphere, such as the annihilation of neutrino pairs,

$$\nu + \bar{\nu} \rightarrow e^+ + e^-$$

are irrelevant.

6. Percentage of neutrinos captured

We are now able to calculate the fraction of neutrinos which are captured at intermediate distances, i.e. between the gain radius R_g and some place like $r =$ 400–500 km.

The energy gain in the region where free nucleons dominate is given by (10). We only need to multiply by $4\pi r^2 \rho\, \mathrm{d}r$, so we get the fraction of neutrino energy absorbed,

$$\alpha = 7 \times 10^{-18} \int \rho\, \mathrm{d}r, \tag{13}$$

where we have inserted A from (8b).

The density can be taken from (5) in which we insert the temperature from (6). Then the percentage of neutrino energy absorbed in the nucleon region is

$$100\alpha_1 = 0.43[1 - T_\alpha^2/T_g^2]^2 (2T_g^2 + T_\alpha^2), \tag{14}$$

where T_α is the temperature at the boundary between nucleon and mixed α-nucleon region. Setting this equal to 1 MeV, we get

$$100\alpha_1 = 1.40. \tag{15}$$

In (14), we have taken into account the energy loss due to electron capture, (9). Equation (14) shows the great sensitivity of the useful energy to the gain temperature T_g, and hence to the neutrino luminosity, see (11).

In the region in which α-particles dominate, the neutrino absorption is proportional to the fraction of free nucleons, and the contribution of this region is

$$100\alpha_2 = 0.73. \tag{15a}$$

Finally, some energy is contributed by collisions of neutrinos with electron and positrons, this is

$$100\alpha_3 = 0.11, \tag{15b}$$

for a total efficiency of 2.14 %.

However, not all the energy deposited in the mantle of the star will go into the shock. As we know, in the convection there is a downstream and an upstream. Part of the material in the downstream (maybe one-half) will accrete, the rest will go into the upstream and hence into the shock. If we guess the accreting part of the downstream to be one-half, it is reasonable to estimate that two-thirds of the energy will go into the shock. Hence our estimate of the net efficiency of neutrino energy absorption is

$$100\alpha_{\mathrm{eff}} = 1.4. \tag{16}$$

7. Energy of supernova

The energy in $\nu_e + \bar{\nu}_e$ in the first 2 s is just two times the luminosity calculated in (7), hence

$$W = 6 \times 10^{52}\ \mathrm{erg}. \tag{17}$$

As we calculated in the last section, 1.4 % of this energy goes to the shock, giving a shock energy of

$$E_1 = 0.8\ \mathrm{foe}$$

(1 foe = 10^{51} erg).

To this, we should add the energy released in nucleosynthesis. Initially, the material outside the final proto-neutron star is mostly ^{16}O. We know that 0.075 $M_\odot$ of this is converted into ^{56}Ni, giving an energy 0.11 foe. It is reasonable to assume that an additional 0.3$M_\odot$ is transformed into ^{28}Si and ^{32}S, a process which releases an additional 0.28 foe. Adding these to the above energy from neutrino absorption gives a total energy

$$E = 1.2\ \mathrm{foe}. \tag{18}$$

Observation of the velocity and mass of the ejecta in SN1987A has given the result

$$E = 1.4 \pm 0.4\ \mathrm{foe}. \tag{19}$$

The agreement is satisfactory.

Using the calculated density in the relevant part of the progenitor of SN1987A, and the observed energy (19), the velocity of the shock is about

$$U = 0.8 \times 10^9 \text{ cm s}^{-1}. \tag{20}$$

8. Developments after explosion

What happens after accretion stops? The neutrino luminosity drops by a factor of about 10, therefore there is no longer a gain radius: the energy loss by electron capture is now greater than the gain by neutrino absorption so the shock wave loses its pressure support, and so its inner portion will fall onto the neutron star, by gravitation.

But this is not true of the outer part of the shock. It is now at a distance greater than 10^9 cm = 10000 km from the centre where gravitation is quite weak. Also it is moving outward with a velocity of order 10000 km s^{-1}. So it will keep moving outward.

I assume that the outward velocity is proportional to the distance from the centre; I have calculated the separation point as the point where the sum of the kinetic energy and the gravitational potential is zero. This puts it at $r = 0.4R_S$ where R_S is the shock radius. Assuming further than the density is uniform in the shocked region, I find that about 0.04 $M_\odot$ falls back onto the neutron star. Since this mass has the smallest kinetic energy, the energy loss from the shock is small.

The compact core may undergo further transformations. G. E. Brown has shown that in nuclear matter of four or more times normal nuclear density, there is condensation of K^- particles. These can replace electrons in providing the negative charge to neutralize the proton charge. This makes the equation of state much softer, and decreases the limiting mass of a neutron star. It is estimated that this limiting (gravitational) mass is about 1.5 $M_\odot$, in accord with all but one of the measured masses of neutron stars. The compact core of a supernova may initially have a mass considerably higher, perhaps 1.7 $M_\odot$. Therefore, it is possible that after the supernova phenomena the compact core may collapse to a black hole. Brown and Bethe have estimated that this may happen to star with initial, main sequence masses of about 18–30 $M_\odot$. Lighter progenitor will leave neutron stars behind. This is compatible with observations of the X-rays from supernova remnants, by Helfand & Becker (1984).

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